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Space/time noncommutativity in string theories without background electric field*

Giuseppe De Risi and Gianluca Grignani

Dipartimento di Fisica and Sezione I.N.F.N., Università di Perugia
Via A. Pascoli I-06123, Perugia, Italy
E-mail: derisi@pg.infn.it grignani@pg.infn.it

Marta Orselli

Dipartimento di Fisica and I.N.F.N. Gruppo Collegato di Parma, Università di Parma
Parco Area delle Scienze 7A I-43100 Parma, Italy
E-mail: orselli@fis.unipr.it

Abstract: The appearance of space/time non-commutativity in theories of open strings with a constant non-diagonal background metric is considered. We show that, even if the space-time coordinates commute, when there is a metric with a time-space component, no electric field and the boundary condition along the spatial direction is Dirichlet, a Moyal phase still arises in products of vertex operators. The theory is in fact dual to the non-commutatitive open string (NCOS) theory. The correct definition of the vertex operators for this theory is provided. We study the system also in the presence of a B field. We consider the case in which the Dirichlet spatial direction is compactified and analyze the effect of these backgrounds on the closed string spectrum. We then heat up the system. We find that the Hagedorn temperature depends in a non-extensive way on the parameters of the background and it is the same for the closed and the open string sectors.

Keywords: Bosonic Strings, D-branes, String Duality.

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Contents

1. Introduction

Noncommutativity in open string theory has been studied since Witten's seminal paper on open string field theory [1]. Recently, there has been much progress in understanding the low energy description of strings and D-branes in electromagnetic backgrounds [2, 3, 4] and how the string dynamics is described by a Yang-Mills field theory with space/space noncommutativity [5]. When D-branes are placed in a background electric field, noncommutativity occurs between time and space coordinates [6, 7]. There is a critical value of the electric field beyond which the theory does not make sense. Since the non-commutative scale is intrinsically tied to the string scale, by approaching the critical field the theory does not become a non-commutative field theory.[1] However, it is possible to consider a particular limit in which the closed strings, and therefore gravity, decouple, leaving a more tractable theory of only open strings. These theories are known as $p+1$ Non Commutative Open String Theories (NCOS) [6, 7], where p is the dimension of the Dp-brane.

It was then argued [13] that when there is a compactified direction, closed strings do not decouple from the spectrum in the NCOS limit. The electric field tends to move apart the open string ends but, since the direction is compactified, they can join with a finite probability after encircling completely the compactified direction and form again a closed string. This explains also the fact that only closed strings with strictly positive winding number are allowed, strings can wind only in one sense. At first sight it may seem strange that the presence of the electric field will change the dynamics of closed strings which are neutral.

[1]Field theories with space/time noncommutativity have inconsistencies related to the lack of unitarity, see for example [8]–[12].

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This is due to the fact that the electric field can be turned into a background Neveu-Schwarz B field by gauge transformations. The argument in [13] was then extended in [14] and in [15], where the NCOS limit of a type-IIA/B superstring theory was considered and a new sector of string theory discovered. These are the Wound String theory of [15] and the Non Relativistic Closed String theory of [14]. These are only closed strings in the absence of D-branes but when D-branes are present represent a generalization of NCOS theories.

In this Paper we shall show that noncommutativity in open string theory can arise also when only a metric with an appropriate form is present and one of the spatial direction has Dirichlet boundary conditions. The theory we consider is dual to the one studied in [6, 7, 13]. In fact, when only a Neveu-Schwarz B-field in the directions B_{0i} is present, the Buscher rules for duality [16, 17, 18] tell us that the dual theory contains only a nontrivial metric and no B field at all. Since there is no B-field the space-time coordinates commute, so how noncommutative effects might arise in this situation is a non-trivial question. We will show that the source of noncommutativity can be found in the Moyal phase which appears in the computation of scattering amplitudes of open string vertex operators. The Moyal phase will now depend on the parameter of the metric which will play the role of the non-commuting parameter. This is the main result of this Paper, it is possible to have a non-commutative string theory also when one has a metric of a particular form and no B-field. The metric background considered admits a NCOS limit in which the closed strings decouple.

When the Dirichlet spatial direction is compactified the theory becomes T-dual to the NCOS considered in [13] and there are finite energy closed string modes with a positive discrete momentum. In this case we give an explanation for the origin of noncommutativity that differs form the one given in [15], where is suggested that the Moyal phase in the T-dual picture emerges as a consequence of a large boost of the system. In our approach the appearance of the Moyal phase is a consequence of the Buscher rules and of the correct definition of the propagators and of the vertex operators in the dual theory.

In section 2 we will perform a canonical analysis for an open bosonic string in the presence of a metric in the 0 and 1 directions, when the direction 1 has Dirichlet boundary conditions. Namely we consider an open string propagating in the background of a D24-brane. Here we will derive the Virasoro generators and the energy spectrum.

In section 3 we will compute the propagators in this theory and define the vertex operators. Taking the operator product expansion of two vertex operators we shall then compute the Moyal phase.

Section 4 is devoted to the analysis of the situation in which both a metric and a B-field are present. We shall show that the Moyal phase depends either on the metric moduli or on B depending on the boundary conditions.

In section 5 we shall consider the situation in which the direction 1 is compactified on a circle of radius R and discuss T-duality.

We shall then revisit the arguments of [13], consider the T-duality relations between different backgrounds and show in which way the NCOS limit for each of them has to be taken. The relation with the DLCQ limit of closed string is discussed. We then consider the case when both the metric and the background B field are present.

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In section 7, we consider the high temperature behavior of the various background we examined, showing that the Hagedorn temperature for open strings depends on the background moduli space. This dependence is the same as the one found for closed string when there is a compactified direction 1.

2. Canonical analysis

We will begin by examining the effect of a certain simple background on the canonical analysis of theories of open bosonic strings. This background is a spacetime metric of the form

$$g_{\mu\nu} = \begin{pmatrix} -1+A^2 & -A & 0 & \dots \\ -A & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \tag{2.1}$$

where A is a constant. To keep the g_{00} component of the metric time-like A must be less than 1, 1 is a critical value for A. The action is

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma g_{\mu\nu} \partial^\alpha X^\mu \partial_\alpha X^\nu\,. \tag{2.2}$$

The boundary conditions we assume are

$$\begin{aligned} \partial_\tau X^1 \mid_{\sigma=0,\pi} &= 0 \\ \left[\left(1-A^2\right)\partial_\sigma X^0 + A\partial_\sigma X^1\right] \mid_{\sigma=0,\pi} &= 0 \\ \partial_\sigma X^a \mid_{\sigma=0,\pi} &= 0 \qquad a = 2,\dots,26 \end{aligned} \tag{2.3}$$

namely the direction 1 is Dirichlet and the transverse directions are Neumann. This background is obtained by a duality transformation [16, 17, 18] from a theory in which the space-time metric is minkowskian, there is a non-zero Neveu-Schwarz B field

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left[\eta_{\mu\nu} \partial^\alpha X^\mu \partial_\alpha X^\nu - \varepsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu\right] \tag{2.4}$$

with

$$B_{\mu\nu} = \begin{pmatrix} 0 & B & 0 & \dots \\ -B & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \tag{2.5}$$

and the boundary conditions are

$$\begin{aligned} \left[\partial_\sigma X^1 + B\partial_\tau X^0\right] \mid_{\sigma=0,\pi} &= 0 \\ \left[\partial_\sigma X^0 + B\partial_\tau X^1\right] \mid_{\sigma=0,\pi} &= 0 \\ \partial_\sigma X^a \mid_{\sigma=0,\pi} &= 0 \qquad a = 2,\dots,26\,. \end{aligned} \tag{2.6}$$

The Buscher rules for this system in fact are

$$g'_{00} = -1+B^2\,, \qquad g'_{01} = B\,, \qquad g'_{11} = 1\,, \qquad B'_{\mu\nu} = 0 \tag{2.7}$$

and lead to a background of the type (2.1) with $A \to -B$.

The variation of the action (2.2) yields the equations of motion and constraints

$$\partial_\alpha \partial^\alpha X^\mu = 0\,, \qquad g_{\mu\nu}\partial_\tau X^\mu \partial_\sigma X^\nu = 0\,. \tag{2.8}$$

The solutions of (2.8) in the background (2.1) with the boundary conditions (2.3) for the 0 and 1 directions are

$$\begin{aligned} X^0(\tau,\sigma) &= x^0 + \sqrt{2\alpha'}\left(\alpha_0^0 + \frac{A}{1-A^2}\alpha_0^1\right)\tau + \sqrt{2\alpha'}\frac{A}{1-A^2}\alpha_0^1\sigma + \\ &\quad + i\sqrt{2\alpha'}\sum_{n\neq 0}\left(\frac{\alpha_n^0}{n}e^{-in\tau}\cos n\sigma + \frac{A}{1-A^2}\frac{\alpha_n^1}{n}e^{-in(\tau+\sigma)}\right) \\ X^1(\tau,\sigma) &= x^1 - \sqrt{2\alpha'}\alpha_0^1\sigma - \sqrt{2\alpha'}\sum_{n\neq 0}\left(\frac{\alpha_n^1}{n}e^{-in\tau}\sin n\sigma\right) \end{aligned} \tag{2.9}$$

whereas the transverse directions have the usual expansions for Neumann coordinates. The conjugate momenta read

$$\begin{aligned} \Pi_0 &= \frac{1}{\pi}\frac{1}{\sqrt{2\alpha'}}\left[-(1-A^2)\left(\alpha_0^0 + \frac{A}{1-A^2}\alpha_0^1\right) + \sum_{n\neq 0} e^{-in\tau}\cos n\sigma\left(-(1-A^2)\alpha_n^0 - A\alpha_n^1\right)\right] \\ \Pi_1 &= \frac{1}{\pi}\frac{1}{\sqrt{2\alpha'}}\left[-A\alpha_0^0 - \frac{A^2}{1-A^2}\alpha_0^1 + \sum_{n\neq 0} e^{-in\tau}\cos n\sigma\left(-A\alpha_n^o - \frac{A^2}{1-A^2}\alpha_n^1\right) + \right. \\ &\quad \left. + \frac{i}{1-A^2}\sum_{n\neq 0} e^{-in\tau}\sin n\sigma\alpha_n^1\right]. \end{aligned} \tag{2.10}$$

The total momentum in the direction 1, which is obtained by integrating in σ the expression for Π_1, is not conserved. This is due to the fact that the presence of the D-brane breaks translational invariance along the direction 1, so it is not possible to define consistently a momentum which is canonically conjugate to the center of mass position x^1. The total momentum P_0 is instead defined in the usual way

$$P_0 = \int_0^\pi d\sigma \Pi_0(\tau,\sigma) = -\frac{1}{\sqrt{2\alpha'}}\left(\left(1-A^2\right)\alpha_0^0 + A\alpha_0^1\right). \tag{2.11}$$

The equal time commutation relations have the standard free field form

$$\begin{aligned} \left[X^\mu(\tau,\sigma), \Pi_\nu(\tau,\sigma')\right] &= i\delta^\mu{}_\nu\delta(\sigma-\sigma') \\ \left[X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')\right] &= \left[\Pi^\mu(\tau,\sigma), \Pi^\nu(\tau,\sigma')\right] = 0\,. \end{aligned} \tag{2.12}$$

These are consistent with the boundary conditions (2.3) and there seems to be no space/time noncommutativity. What happens here is quite different to the dual case [20], where the boundary conditions (2.6) are inconsistent with the canonical commutation relations and noncommutativity appears already at this stage.

In order to obtain the relations (2.12), the oscillation modes must satisfy the standard commutation relations

$$[\alpha_n^\mu, \alpha_m^\nu] = n\delta_{n+m,0}g^{\mu\nu}\,, \qquad n, m \neq 0 \tag{2.13}$$

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and the commutators between the total momenta P_0 and P_a $a = 2, \ldots, 26$ and the center of mass modes x^μ are as expected

$$[x^a, P_b] = i\delta^a{}_b\,, \qquad [x^0, P_0] = i\,, \qquad [x^1, P_0] = 0\,. \tag{2.14}$$

The commutation relations (2.13), (2.14) are just enough to satisfy the canonical commutators, but something seems to be missing. In fact in these commutation relations there is no information about the degree of freedom related to the zero mode of the direction 1. This operator should be proportional to α_0^1. To construct the complete Fock space of the string, instead of the canonical momentum P_1, it is then necessary to introduce the operator

$$L^1 = \frac{1}{\sqrt{2\alpha'}}\alpha_0^1 \tag{2.15}$$

and the generic state of the string should be represented as $|N, k_0, l^1, \vec{k}\rangle$ where l^1 is the eigenvalue of L^1. It is easy to see that this operator commutes with the other canonical momenta. The physical interpretation of this operator becomes clear when the direction 1 is compactified. Its eigenvalue is proportional to the winding number of the open string around the compact direction. We shall come back to this in section 5.

To construct the vertex operators it is necessary to find the variable Q^1 which is canonically conjugate to L_1, *i.e.* $[Q^1, L_1] = i$. For this purpose we invoke the doubling formalism to solve the equations of motion (2.8). In fact the most general solution of (2.8) is the sum of the left, X_L^1, and right, X_R^1, mover modes

$$\begin{aligned} X_L^1(\tau-\sigma) &= x_L^1 + \sqrt{\frac{\alpha'}{2}}\,\alpha_0^1(\tau-\sigma) + \text{ osc.} \\ X_R^1(\tau+\sigma) &= x_R^1 + \sqrt{\frac{\alpha'}{2}}\,\tilde{\alpha}_0^1(\tau+\sigma) + \text{ osc.} \end{aligned} \tag{2.16}$$

thus leading to the definition of a left and a right momentum $p_L^1 = \sqrt{\frac{2}{\alpha'}}\alpha_0^1$ and $p_R^1 = \sqrt{\frac{2}{\alpha'}}\tilde{\alpha}_0^1$. Taking into account the boundary conditions (2.3) it is easy to see that $\alpha_0^1 = -\tilde{\alpha}_0^1$, so that the operator (2.15) is given by the difference $L^1 = \left(p_L^1 - p_R^1\right)/2$. From this we can argue [21, 22] that the variable canonically conjugate to L^1 is given by the difference of the constant modes of the left and right expansion (2.16)

$$Q^1 = x_L^1 - x_R^1\,. \tag{2.17}$$

In what follows we shall also need the energy spectrum. This can be easily derived from the Virasoro generators

$$L_m = \sum_{n\in Z} g_{\mu\nu}\alpha_{m-n}^\mu \alpha_n^\nu \tag{2.18}$$

and the condition $L_0 - 1 = 0$. This yields

$$k_0 = \sqrt{(l^1)^2 + (1-A^2)\left[\vec{k}^2 + \frac{(N-1)}{\alpha'}\right]} \tag{2.19}$$

where k_0 and l^1 are the eigenvalues of P_0 and L^1, respectively. $\vec{k}$ is the transverse momentum and $N = \sum_{n=1}^\infty \alpha_{-n}\cdot\alpha_n$ with a standard notation for oscillators [23].

3. Evaluation of the moyal phase

In this section we compute the product of two tachyon vertex operators and show how the space/time noncommutativity manifests itself: the product of two vertex operators will give rise to a Moyal phase, namely it can be interpreted as a Moyal $*$ product. We will work at tree level in string theory and represent the string worldsheet as the upper half plane using the convention of ref. [24].

Consider the propagator of the theory in the background (2.1) and with boundary conditions (2.3)

$$\langle X^\mu(z_1) X^\nu(z_2)\rangle = -\frac{\alpha'}{2}\left[g^{\mu\nu}\log|z_1-z_2|^2 - A^{\mu\nu}\log|z_1-\bar{z}_2|^2 + D^{\mu\nu}\right] \tag{3.1}$$

where

$$g^{\mu\nu} = \begin{pmatrix} -1 & -A & 0 & \dots \\ -A & 1-A^2 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}, \qquad A^{\mu\nu} = \begin{pmatrix} \frac{1+A^2}{1-A^2} & -A & 0 & \dots \\ -A & 1-A^2 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}. \tag{3.2}$$

The constants $D^{\mu\nu}$ are independent on z_1 and z_2 and can be set to a convenient value. The propagator (3.1) is symmetric so that the boundary propagator will not contain a noncommutativity parameter. By interpreting time ordering as operator ordering the commutator $[X^0(\tau), X^1(\tau)]$ will just vanish. The antisymmetric contribution responsible for the Moyal phase in the NCOS theories [5] seems to be absent.

We must now find an appropriate expression for the vertex operators. They should posses the same symmetry of the correspondent states both under translation and under the symmetry generated by the operator (2.15), which can be viewed as a translation of the left and right parts of X^1 in opposite directions. This is consistent with the fact that the dual symmetry reverses the sign of the right part of the coordinate: $\tilde{X}^1(z,\bar{z}) = X^1_L(z) - X^1_R(\bar{z})$. Since the operator L_1 commutes with the other operators that generate the Fock space, its eigenvalue is preserved throughout any interaction and the vertex operator should depend on it. Consequently, for the tachyon vertex operator one can write [21]

$$\mathcal{V}_{k_0,w,\vec{k}} =: e^{ik_0 X^0(z,\bar{z}) + il_1(X^1_L(z) - X^1_R(\bar{z})) + ik_a X^a(z,\bar{z})} : \tag{3.3}$$

where k_0, l_1 and k_a are the eigenvalue of P_0, $L_1 = \alpha_0^1/2\alpha'(1-A^2)$ and of the transverse momenta P_a.

The world-sheet integral of the vertex operator must be conformally invariant. This implies that (3.3) must be a tensor of weight (1/2,1/2). By a simple OPE computation, it is possible to show that the vertex (3.3) is a tensor of weight

$$h = \bar{h} = -\frac{\alpha'}{2\,(1-A^2)}\left[k_0^2 - (l^1)^2 - \left(1-A^2\right)\vec{k}^2\right].$$

This gives for the tachyon

$$k_0 = \sqrt{(l^1)^2 + (1-A^2)\left(\vec{k}^2 - \frac{1}{\alpha'}\right)} \tag{3.4}$$

which agrees with the expression derived from the constraint $L_0 - 1 = 0$. This again confirms the validity of the expression (3.3) for the vertex operator.

Since the vertex (3.3) depends on the holomorphic and anti-holomorphic parts of $X^1(z_1, \bar{z}_1) = X^1_L(z) + X^1_R(\bar{z})$ it is useful to explicitly write the propagators of these modes separately. These can be obtained from (3.1) by keeping into account that $X_L(z)$ is holomorphic and $X_R(\bar{z})$ anti-holomorphic. The relevant propagators are

$$\begin{aligned}
\langle X^0(z_1) X^1_L(z_2)\rangle &= \frac{\alpha'}{2} A \log \frac{z_1 - z_2}{\bar{z}_1 - z_2} \\
\langle X^0(z_1) X^1_R(\bar{z}_2)\rangle &= \frac{\alpha'}{2} A \log \frac{\bar{z}_1 - \bar{z}_2}{z_1 - \bar{z}_2} \\
\langle X^1_L(z_1) X^1_L(z_2)\rangle &= -\frac{\alpha'}{2}\left(1 - A^2\right) \log\left(z_1 - z_2\right) \\
\langle X^1_R(\bar{z}_1) X^1_R(\bar{z}_2)\rangle &= -\frac{\alpha'}{2}\left(1 - A^2\right) \log\left(\bar{z}_1 - \bar{z}_2\right) \\
\langle X^1_L(z_1) X^1_R(\bar{z}_2)\rangle &= \frac{\alpha'}{2}\left(1 - A^2\right) \log\left(z_1 - \bar{z}_2\right)
\end{aligned} \tag{3.5}$$

which again are defined up to a constant.

Consider now the OPE for the product of two vertex operators inserted on the boundary of the worldsheet, $\mathcal{V}_1(\tau)\mathcal{V}_2(0)$. To evaluate this OPE consider the propagators (3.1) and (3.5) at the boundary points τ and 0. We have

$$\begin{aligned}
\langle X^0(\tau) X^0(0)\rangle &= \frac{\alpha'}{1 - A^2} \log \tau^2 \\
\langle X^0(\tau)\left(X^1_L(0) - X^1_R(0)\right)\rangle &= -i\alpha' \pi A \epsilon(\tau) \\
\langle\left(X^1_L(\tau) - X^1_R(\tau)\right)\left(X^1_L(0) - X^1_R(0)\right)\rangle &= \alpha'\left(1 - A^2\right) \log \tau^2
\end{aligned} \tag{3.6}$$

where $\epsilon(\tau)$ is the function that is 1 or -1 for positive or negative τ and the constant $D^{\mu\nu}$ has been set to a convenient value. With the boundary propagators (3.6) the product of two normal ordered tachyon vertex operators satisfy (we ignore the transverse coordinates)

$$\begin{aligned}
&: e^{ik_0 X^0(\tau) + il_1\left(X^1_L(\tau) - X^1_R(\tau)\right)} :: e^{ik'_0 X^0(0) + il'_1\left(X^1_L(0) - X^1_R(0)\right)} := \\
&\quad = \exp\left\{-\frac{\alpha'}{1 - A^2}\left(k_0 k'_0 - l^1 l'^1\right) \log \tau^2 + i\pi\alpha' \epsilon(\tau) \frac{A}{1 - A^2}\left(k_0 l'^1 - k'_0 l^1\right)\right\} \times \\
&\quad \times : e^{ik_0 X^0(\tau) + il_1(X^1_L(\tau) - X^1_R(\tau)) + ik'_0 X^0(0) + il_1(X^1_L(0) - X^1_R(0))} :
\end{aligned} \tag{3.7}$$

where $l^1 = (1 - A^2) l_1$ is the quantity dual to the momentum in the direction 1, k_1, of the dual theory. The form of the OPE (3.7) is precisely dual to the well known expression derived in [5] and in the $\alpha' \to 0$ limit defines a Moyal product with non commutative parameter given by

$$\theta = 2\pi\alpha' \frac{A}{(1 - A^2)}\,. \tag{3.8}$$

The Moyal phase is

$$\exp\left[i\pi\alpha' \frac{A}{(1 - A^2)}\left(k_0 l'^1 - k'_0 l^1\right)\right]. \tag{3.9}$$

In (3.7) the short distance singularity is governed by the "open string metric"

$$G^{\mu\nu} = \frac{1}{1 - A^2} \begin{pmatrix} -1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 1 - A^2 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \tag{3.10}$$

and determines the anomalous dimension of the vertex operators.

Now let $A = 1 - \epsilon/2$. We can define a NCOS type limit as

$$\epsilon \to 0\,, \quad \text{with} \quad \alpha' = \alpha'_e \epsilon\,, \quad \vec{K} = \vec{k}\epsilon\,; \quad \alpha'_e \quad \text{fixed}\,. \tag{3.11}$$

In this limit the open string spectrum (2.19) becomes

$$k_0 = \sqrt{(l^1)^2 + \left[\vec{K}^2 + \frac{(N-1)}{\alpha'_e}\right]}. \tag{3.12}$$

and remains finite. As in [6] the closed string spectrum instead diverges and closed strings decouple. The anomalous dimension of the vertex operator could be ignored and the Moyal phase becomes

$$\exp\left[i\pi\alpha'_e\left(k_0 l'^1 - k'_0 l^1\right)\right] \tag{3.13}$$

4. Inclusion of the B field

Next we may add a constant Neveu-Schwarz B-field to the action. It becomes

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left[g_{\mu\nu}\partial^\alpha X^\mu \partial_\alpha X^\nu - \varepsilon^{\alpha\beta}B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right]. \tag{4.1}$$

The B-dependent term in the action (4.1) is a total derivative, so it does not affect the equations of motion. Since the direction 1 is Dirichlet the boundary conditions are still given by (2.3). Consequently, also the propagators are left unchanged. However, the presence of the B field affects the energy spectrum by shifting its value by a constant:

$$k_0 = Bl^1 + \sqrt{(l^1)^2 + (1 - A^2)\left[\vec{k}^2 + \frac{(N-1)}{\alpha'}\right]} \tag{4.2}$$

It is interesting to notice that contrary to what happens in NCOS theories, in this case the antisymmetric tensor field $B_{\mu\nu}$ does not participate to the Moyal phase (3.9). B affects only P_0 as in (4.2), and, being the propagators unchanged, the phase keeps the form (3.9). In fact, since the eigenvalue k_0 is shifted by Bl^1, the B dependent terms cancel.

In the same way it is possible to show that starting with the dual theory with both the metric and the B field, even if the metric modifies the energy spectrum of the theory, it does not affect the Moyal phase. Let us consider this case in more detail. The action is again (4.1) but the boundary conditions now are

$$\begin{aligned} \left[g_{00}\partial_\sigma X^0 + g_{01}\partial_\sigma X^1 - B\partial_\tau X^1\right]|_{\sigma=0,\pi} &= 0 \\ \left[g_{10}\partial_\sigma X^0 + g_{11}\partial_\sigma X^1 \partial_\sigma X^0 + B\partial_\tau X^0\right]|_{\sigma=0,\pi} &= 0 \\ \partial_\sigma X^a\,|_{\sigma=0,\pi} &= 0 \qquad a = 2, \dots, 26\,. \end{aligned} \tag{4.3}$$

The equations of motion and constraints are given by (2.8) and, taking into account the boundary conditions (4.3), have the solutions

$$\begin{aligned}
X^0(\tau,\sigma) &= x^0 + 2\alpha' p^0\tau + 2\alpha' ABp^0\sigma - 2\alpha' Bp^1\sigma + \\
&\quad + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha_n^0\cos n\sigma + AB\alpha_n^0\sin n\sigma - B\alpha_n^1\sin n\sigma\right)\\
X^1(\tau,\sigma) &= x^1 + 2\alpha' p^1\tau - 2\alpha' ABp^1\sigma + 2\alpha'\left(A^2-1\right)Bp^0\sigma\\
&\quad + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left[i\alpha_n^1\cos n\sigma - AB\alpha_n^1\sin n\sigma + \left(A^2-1\right)B\alpha_n^0\sin n\sigma\right]
\end{aligned} \tag{4.4}$$

and the usual expansions hold for the transverse Neumann coordinates. Following [20] the commutation relations of the modes in (4.4) are

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}\,, \quad [x^\mu, p^\nu] = iM^{-1\mu\nu}\,, \quad [\alpha_m^\mu, \alpha_n^\nu] = im\delta_{m+n,0}M^{-1\mu\nu} \tag{4.5}$$

where

$$M_{\mu\nu} = g_{\mu\nu} - B_{\mu\rho}B^\rho{}_\nu\,, .\theta^{\mu\nu} = 2\pi\alpha'\frac{B^{\mu\nu}}{1-B^2} \tag{4.6}$$

From the action (4.1) the canonical momentum is

$$P^\mu = \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma\left(\partial_\tau X^\mu + \partial_\sigma X^\nu B_\nu{}^\mu\right) = p^\nu M_\nu{}^\mu\,. \tag{4.7}$$

The momentum P^μ is conjugate in the usual sense to the center of mass coordinate $x^\mu_{\text{c.m.}}$ defined as

$$x^\mu_{\text{c.m.}} = \frac{1}{\pi}\int_0^\pi d\sigma X^\mu(\tau,\sigma)\,.$$

By imposing the equation (2.8) on the physical states one can derive the Virasoro generators [20]

$$L_m = \sum_{n\in Z} M_{\mu\nu}\alpha^\mu_{m-n}\alpha^\nu_n \tag{4.8}$$

so that the energy spectrum reads

$$k_0 = -Ak_1 + \sqrt{k_1^2 + (1-B^2)\left[\vec{k}^2 + \frac{(N-1)}{\alpha'}\right]}. \tag{4.9}$$

This energy spectrum is dual to (4.2), it becomes (4.2) when $A \leftrightarrow -B$ and the momentum along the direction 1 is substituted with the eigenvalue of the operator L^1.

The vertex operator in this case has the usual form

$$\mathcal{V}_{k_\mu} =: e^{ik_\mu X^\mu(z,\bar{z})} :\ . \tag{4.10}$$

With the boundary conditions (4.3) the propagators read

$$\begin{aligned}
\langle X^\mu(z_1)X^\nu(z_2)\rangle &= -\frac{\alpha'}{2}\Big[g^{\mu\nu}\log|z_1-z_2|^2 - g^{\mu\nu}\log|z_1-\bar{z}_2|^2 + \\
&\quad + 2G^{\mu\nu}\log|z_1-\bar{z}_2|^2 + \frac{1}{\pi\alpha'}\theta^{\mu\nu}\log|z_1-\bar{z}_2|^2 + D^{\mu\nu}\Big]
\end{aligned} \tag{4.11}$$

where $g^{\mu\nu}$ is the metric (3.2) and

$$G_{\mu\nu} = g_{\mu\nu} - \left(Bg^{-1}B\right)_{\mu\nu}$$

is the open string metric.

In the OPE of two normal ordered vertex operators a Moyal phase with the form

$$\exp\left[-i\pi\alpha'\frac{B}{(1-B^2)}\left(k_0k_1' - k_0'k_1\right)\right] \tag{4.12}$$

appears. It does not depend on A and is dual to (3.9).

5. The compact case

In this section we will compactify the direction 1.

$$x^1 \sim x^1 + 2\pi R\,. \tag{5.1}$$

The duality symmetry discussed in the previous sections becomes now T-duality. The origin of non-commutativity in T-dual NCOS has raised some discussions [15, 25], here we give a different interpretation.

Consider first the case described by the action (2.2) with boundary conditions (2.3). Being the direction 1 compactified we must require

$$\alpha_0^1 = \sqrt{\frac{2}{\alpha'}}Rw \tag{5.2}$$

where w is an integer. As a matter of fact an open string with Dirichlet boundary conditions along a compact dimension can have non-trivial winding modes. Since the ends of the string are tied to the D-brane it cannot unwrap. On the other hand the Dirichlet string does not have Kaluza Klein momenta.

We may now clarify the physical meaning of the operator L^1. When the direction 1 is compactified, it becomes exactly the operator associated with the winding modes of the string. So, in the decompactified limit we find a sort of “continuum winding”, described by the eigenvalue l^1. All the computations above are unchanged, but one must set

$$l^1 = \frac{wR}{\alpha'}\,. \tag{5.3}$$

In particular the Moyal phase becomes

$$\exp\left[i\pi R\frac{A}{(1-A^2)}\left(k_0w' - k_0'w\right)\right]. \tag{5.4}$$

This expression is manifestly T-dual to (4.12), where, due to the compactification, k_1 is quantized as $k_1 = m/R$.

6. Closed strings

In [13] it was argued that when there is a compactified direction, wound states of closed strings do not decouple from the spectrum in the NCOS limit. These closed string states were used to construct Wound String theory in [15] and Non-Relativistic Closed Strings [14]. In this section we will see how these arguments apply to our situation. For closed strings the canonical momenta in the compactified direction is quantized

$$k_1 = \frac{m}{R}, \qquad m \in Z \tag{6.1}$$

the energy spectrum for closed string when only the metric (2.1) is present reads

$$k_0 = -\frac{Am}{R} + \sqrt{\left(\frac{wR}{\alpha'}\right)^2 + \left(\frac{m}{R}\right)^2 + \vec{k}^2 + \frac{2}{\alpha'}(N + \tilde{N} - 2)} \tag{6.2}$$

where $\vec{k}$ denotes the transverse momentum, $N = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$ and $\tilde{N} = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n$. Let $A \sim 1 - \varepsilon/2$ and define the relevant constant for the limiting process (notice that in the usual description R is held fixed)

$$\alpha'_e = \frac{\alpha'}{\varepsilon}, \qquad \vec{k}^2 = \frac{\vec{K}^2}{\varepsilon}, \qquad R_e = \frac{R}{\varepsilon}. \tag{6.3}$$

The NCOS limit [6, 7] consists of taking $\varepsilon \to 0$, keeping the constants (6.3) fixed. The spectrum of closed string diverges in this limit, unless $m > 0$, and in this case we obtain:

$$k_0 = \frac{m}{2R_e} + \frac{R_e}{2m}\vec{K}^2 + \frac{R_e}{\alpha'_e m}(N + \tilde{N} - 2). \tag{6.4}$$

The result that k_1 must be positive is an indication of the T-duality relation between NCOS theories and the DLCQ strings [13, 15]. In fact, defining $k^+ = \sqrt{2}k^0 - k^-$ and $k^- = m/(\sqrt{2}R_e)$ we obtain exactly the DLCQ closed string spectrum [26, 27]

$$k^+ = \frac{N + \tilde{N} - 2}{\alpha'_e k^-} + \frac{\vec{K}^2}{2k^-}. \tag{6.5}$$

Notice also that the expression (6.4) is T-dual to the spectrum obtained in [15] with the parameter (6.3) of the NCOS theory and taking into account that in the dual situation $R = R_e$.

By adding the B field one changes the spectrum according to

$$k_0 = \frac{BRw}{\alpha'} - \frac{Am}{R} + \sqrt{\left(\frac{wR}{\alpha'}\right)^2 + \left(\frac{m}{R}\right)^2 + \vec{k}^2 + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)}. \tag{6.6}$$

It is now possible to take the NCOS limit by letting $B = 1 - \varepsilon/2$ and by keeping α'_e, the rescaled transverse momentum $\vec{K}^2$ and the radius constant as in [15], thus obtaining

$$k_0 = -\frac{Am}{R} + \frac{wR}{2\alpha'_e} + \frac{\alpha'_e}{2wR}\vec{K}^2 + \frac{N + \tilde{N} - 2}{wR} \tag{6.7}$$

where $w > 0$. Otherwise one can put $A = 1 - \varepsilon/2$ and by keeping constant the parameter (6.3) in the limit $\epsilon \to 0$ the spectrum reads

$$k_0 = \frac{BR_e w}{\alpha'_e} + \frac{m}{2R_e} + \frac{R_e}{2m}\vec{K}^2 + \frac{R_e}{\alpha'_e m}(N + \tilde{N} - 2) \tag{6.8}$$

with $m > 0$. Of course this two cases are related by T-duality, with $R = R_e$ in the first case, and from both it is easy to derive the DLCQ spectrum for closed string in the presence of a background B or A field.

7. The hagedorn transition

To better understand the physics of the strings in the background we are considering it is useful to subject them to extreme conditions by heating them up to high temperatures. For conventional superstring theory this has been extensively studied (see for instance [28]–[31]). In the presence of spacetime backgrounds the Hagedorn temperature was studied for example in refs. [32, 33, 34, 19]. The systems studied in these papers are gravitating so that it is difficult to study their thermodynamics. Nevertheless the Hagedorn transition has been interpreted as a first order phase transition [30]. Since the NCOS are decoupled from gravity their thermodynamics can be analyzed in detail and the phase diagram has an extremely rich structure [35, 36, 37]. It also has gauge theory analogs as shown in [38]. The NCOS transition becomes of second order and can be studied in the context of weakly coupled string theory. However, when a direction is compactified wrapped states of closed strings do not decouple from the spectrum in the NCOS limit [13]. One would expect that, in the limit as the compactification radius is large, the wrapped closed strings would couple more and more weakly and in the infinite, de-compactified limit they would disappear from the spectrum. Indeed their energies do go to infinity. However it was shown in [19] that their Hagedorn temperature remains, that is, no matter how large that radius is, they still participate in the Hagedorn transition. It was argued in [19] that the closed string Hagedorn behavior makes it a first order transition again.

In this section we will investigate the thermodynamic properties of the open string sector in the backgrounds considered above showing that the Hagedorn temperature is modified by the presence of the backgrounds. Finally, we will compactify the direction 1 and we will show that for the open string sector the dependence of the Hagedorn temperature on the background moduli has the same non-extensive (radius independent) behavior as that of the closed string sector [19]. The two sectors undergo a phase transition at the same time.

Consider first the case in which the background is only metric (2.1) with boundary conditions (2.3).

The free energy of a gas of relativistic Bose particles is

$$F = \frac{1}{\beta}\mathrm{Tr}\ln\left(1 - e^{-\beta P_0}\right) = -\sum_{n=1}^{\infty}\frac{1}{n\beta}\mathrm{Tr}e^{-n\beta P_0}\,. \tag{7.1}$$

As well known, (7.1) can be used to derive the bosonic string free energy at one loop, by the standard procedure of computing the sum of free energies of the particles in the string spectrum. The energy spectrum is given by equation (2.19).

To obtain the free energy of the bosonic string we use the integral identity

$$\int_0^\infty dt e^{-xt^2-y/t^2} = \frac{1}{2}\sqrt{\frac{\pi}{x}} e^{-2\sqrt{xy}}$$

where

$$t^2 = \frac{1}{\tau_2}, \qquad x = \frac{n^2\beta^2}{4\pi\alpha'}\left(1-A^2\right), \qquad y = \pi\alpha'\left[\frac{(l^1)^2}{(1-A^2)} + \vec{P}^{\,2} + \frac{1}{\alpha'}(N-1)\right].$$

Then the free energy reads

$$F = -\sum_{n=1}^{\infty}\int_0^\infty \frac{d\tau_2}{\tau_2}\frac{1}{(4\pi^2\alpha'\tau_2)^{\frac{D}{2}}}\left|\eta\left(\frac{\tau_2}{2}\right)\right|^{-24} e^{-\frac{\beta^2 n^2}{4\pi\alpha'\tau_2}(1-A^2)-\pi\alpha'\tau_2\frac{(l^1)^2}{(1-A^2)}}. \tag{7.2}$$

The temperature independent $n = 0$ term gives the vacuum energy, *i.e.* the cosmological constant contribution, the other terms give the relevant thermodynamic potential. The Hagedorn temperature is by definition the temperature at which the one-loop free energy (7.2) diverges. This happens for $\tau_2 \to 0$. In this limit it is useful to write the Dedekind eta function in term of a series as in [23]

$$\left|\eta(\frac{\tau_2}{2})\right|^{-24} = e^{\pi\tau_2}\left|\prod_{m=1}^{\infty}\left(1-e^{-\pi\tau_2 m}\right)\right|^{-24} \tag{7.3}$$

and

$$\prod_{d=1}^{\infty}\left(1-e^{-\pi\tau_2 m}\right)^{-24} \equiv \sum_{r=0}^{\infty} d(r) e^{-\pi\tau_2 r}. \tag{7.4}$$

For large r one gets [39]

$$d(r) \sim r^{-27/4} e^{4\pi\sqrt{r}}. \tag{7.5}$$

In the $\tau_2 \to 0$ limit the sums are dominated by those integers for which r is big. Moreover, the dominant term is obtained by setting $n = 1$. Then for $\tau_2 \sim 0$ we could use a saddle point procedure for the variable r to evaluate the sum

$$\sum_{r=0}^{\infty} r^{-27/4} e^{4\pi\sqrt{r}-\pi\tau_2 r}. \tag{7.6}$$

The saddle point equation has the solution

$$\sqrt{r} = \frac{2}{\tau_2}. \tag{7.7}$$

Substituting the solution in the expression of the free energy and taking the $\tau_2 \to 0$ limit, we find that the Hagedorn temperature is

$$T = T_H\sqrt{(1-A^2)} \tag{7.8}$$

where $T_H = \frac{1}{4\pi\sqrt{\alpha'}}$ is the Hagedorn temperature in the absence of the background metric. The result (7.8) is quite expected since it is exactly the NCOS behavior but with B replaced by $-A$.

We now turn to examine the Hagedorn temperature when both the metric (2.1) and the constant Neveu-Schwarz B-field (2.5) are present. The boundary conditions we choose are (2.3).

The string energy spectrum is modified by the presence of the B-field and is given by equation (4.2). In this case the expression for the free energy becomes

$$F = -\sum_{n=1}^{\infty} \int_0^{\infty} \frac{d\tau_2}{\tau_2} \frac{\sqrt{1-A^2}}{(2\pi\alpha'\tau_2)^{\frac{D}{2}}} \left|\eta\left(\frac{\tau_2}{2}\right)\right|^{-24} e^{-\frac{\beta^2 n^2}{4\pi\alpha'\tau_2}(1-A^2)+\frac{\beta^2 n^2}{4\pi\alpha'\tau_2}B^2(1-A^2)}. \tag{7.9}$$

We can then proceed as before and use a saddle point procedure for the variable r. Substituting the solution of the saddle point equation in the expression of the free energy one can easily see that the exponent vanishes when

$$T = T_H\sqrt{(1-A^2)(1-B^2)} \tag{7.10}$$

where again T_H is the Hagedorn temperature for the open bosonic strings in the absence of the background metric and antisymmetric tensor field. The Hagedorn temperature is self-dual when the duality transformation is along the spatial direction. Whereas the Moyal phase depends only on one of the two background moduli, the Hagedorn temperature depends on both. By studying the high temperature behavior of the theory with boundary conditions (4.3) we obviously find for the Hagedorn temperature the same result (7.10), the energy spectra are in fact dual.

The Hagedorn temperature can be determined also when the direction 1 is compactified, it is easy to see that also in this case the result (7.10) still holds and it coincides with the one obtained for the closed string sector [19]. The formula (7.10) in this case has a remarkable feature. It depends on A and B, but for fixed A and B, it does not depend on the compactification radius R. The physical picture of what is happening has been given in ref. [19], here we review the argument. When the boundary conditions are (2.3), there is a region of the parameter space where A and B are between 0 and 1, away from their limiting values and where R is very large so that all wrapped states have a very large energy. In that case, at temperatures just below T_H, practically no wrapped states are excited in the thermal distribution. However, since T_H depends on B, it must be wrapped states which condense at the Hagedorn transition, in fact the resulting long string must wrap the compact dimension. Thus we see that, in the limit where R is very large, when the temperature T_H is reached, there is a catastrophic process where dominant configurations in the ensemble go from a thermal distribution of multi-string states with zero wrapping to a single long string which wraps the compact dimension. The same considerations are valid in the T-dual theory with boundary conditions (4.3) provided the winding number is exchanged with the quantum of the momentum in the compactified direction.

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In the thermal ensemble the total energy is proportional to the volume, there is sufficient energy to produce strings of arbitrary length whose energy only scale like their length. Then, the R-dependence of the total energy, which grows linearly in R if the temperature is held fixed, is similar to the energy dependence of a wrapped string which also scales linearly with R. There is always enough energy for a long string to wrap the compactified direction no matter how large is the compactification radius.

In [13] it was noted that, when the compactified dimension has finite radius, the wrapped closed string states do not decouple in the NCOS limit. These wrapped states get infinitely large energy in the limit where the radius of the compact dimension is taken to infinity. However, since the Hagedorn temperature does not depend on the compactification radius, the closed strings still participate in the Hagedorn transition. We see that, if the radius is very large but finite, the phase transition for the NCOS, which is believed to be a second order phase transition, becomes first order for the presence of the closed strings in the spectrum.

8. Conclusions

In this Paper, we have studied the origin of space/time noncommutativity in open string theory. We have shown that when an open bosonic string propagates in a background metric of the form (2.1) and with boundary conditions (2.3), a Moyal phase arises when calculating the OPE of vertex operators. The OPE needs a careful derivation of the propagators of the theory and the correct definition of the vertex operators. The vertex operators are elements of an algebra of function defined with the Moyal $*$ product [5] instead of the usual commutative product.

In our approach the noncommutativity is not related to the presence of the antisymmetric tensor field $B_{\mu\nu}$, but it depends only on the metric moduli space.

We also study the effect of the background considered on the Hagedorn temperature showing that it depends on both the parameters of the background metric and the antisymmetric tensor field. The same dependence on the backgrounds is valid for the open and closed string sector when the Dirichlet spatial direction is compactified.

Recently there has been some attempts to study how noncommutativity arises when more general backgrounds are considered. In [40] a time dependent noncommutativity parameter was computed in a model with a time-dependent background. In [41] it was shown that starting with type-IIB string theory on the pp-wave background with a compact lightlike direction and performing a T-duality over the lightlike direction, one can go to a type-IIA description in terms of a non-relativistic closed string theory (NRCS) which is tightly related to the model studied in this Paper. It would be interesting to study these more general cases in view of our results.

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